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Simplified dispersion analysis based on dye tests at a small stream

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Abstract: The modelling of solid transport in open channels requires good knowledge about parameters related to basic processes such as hydrodynamic dispersion, advection and decay rates. Such parameters are usually determined by dye tests. Numerous tracer studies have been performed on laboratory flumes and natural rivers. However, on-site sampling is often difficult, expensive and needs special apparatus. The main aim of the study was to justify simplified method based on the monitoring of the dye cloud shape in order to determine both longitudinal and transversal dispersion coefficients. In this study, four dye tests were carried out on a small local stream (the Lipkovsky) using Rhodamine WT fluorescein dye as a tracer. The tests were carried out in such a manner that both longitudinal and horizontal transversal dispersion data were obtained. For this purpose, the visually determined extent of the dye cloud was interpreted via the analytical solution of the advection-dispersion equation. The results obtained by this simplified approach indicated that the longitudinal dispersion coefficient D_x = 0.051–0.057 m^2/s and the coefficient of horizontal transversal dispersion D_y = 0.00024–0.00027 m^2/s. The method was justified by corresponding root mean square error (RMSE) counting RMSE = 0.65–1.02 m for the dye cloud centre, RMSE = 1.87–2.46 m for the head and tail of the cloud and RMSE = 0.025–0.11 m for the cloud width, the Nash-Sutcliffe efficiency coefficients ranged from 0.9 to 0.998. The comparison of these values with empirical formulae and other tracer studies indicated significant overestimation of the mentioned values of D_x, which can be attributed to the uniform velocity distribution along the width of Lipkovsky Stream. Much better agreement was achieved for D_y.

Keywords: Water quality modelling; 2D river mixing; Dye test; Longitudinal and transversal dispersion coefficient.

1. INTRODUCTION

Stream water quality modelling has been widely used since the 1970´s to assess current conditions and the impacts of proposed measures for water quality improvement in open channels (Brown and Barnwell, 1987; Chapra, 1997; Crowder et al., 2004; DHI, 2010; HEC-RAS, 2022). The application of models requires reliable data on parameters describing transport processes such as hydrodynamic dispersion, advection and decay rates. Advection phenomena can be described by standard hydrodynamics. The reliability of numerical models primarily depends on their input parameters, which include characteristics such as the geometry of channel and hydraulic structures, channel roughness, and transport characteristics, of which the most influential is hydrodynamic dispersion. Also, the implementation of initial and boundary conditions is very important. Even if the values of the mentioned parameters may be determined using various predictive techniques like genetic programming (Azamathulla and Wu, 2011; Riahi-Madvar et al., 2009) or empirical formulae, the most reliable technique is considered to be backward analysis via the calibration of numerical models (Ani et al., 2009) using data from tracer studies. Even if available literature provides numerous results of tracer studies, new data are still needed for particular cases such as small or winding streams with various hydraulic characteristics. The realisation and evaluation of dye tests is often limited by the unavailability of sampling and appropriate measuring devices by which only point samples of the dye may be taken and processed. Halmová et al. (2014) summarized the results of field experiments in small streams and assessed the coefficient of longitudinal dispersion. Results from salt experiments at different flows for various hydrological and vegetation conditions resulted in the coefficients of longitudinal dispersion in the range from 0.2 to 2.5 m^2/s. Shin et al. (2020) used the velocity and concentration information collected from tracer tests in large-scale meandering channels and calculated the longitudinal and transverse dispersion coefficients using the two-dimensional advection-dispersion model. Park et al. (2020) evaluated two different forms of spatially changing dispersion tensors employing both vertical velocity profiles and depth-averaged flow fields which enabled to estimate the arrival time and peak concentration and to compare both approaches used.

Dye tests (sometimes referred as tracer experiments) have been carried out in open channels of various sizes to derive dispersion coefficients and verify empirical formulae (Kim, 2012; Leibundgut et al., 1993; Uyigue and Abah, 2020; Van Mazijk, 1996; Veliskova and Kohutiar, 1992). Nadal et al. (2021) investigated and compared the values of the longitudinal dispersion coefficient obtained by using two different methodologies. The first method involved applying a formula that has been created that contains a thorough description of the hydrodynamic parameters measured with a hydroacoustic device, while the second method involved injecting a conservative tracer using the same methodology as the non-ideal chemical reactor theory of flow with dispersion. Evaluation of longitudinal dispersion coefficients via inverse analysis usually involves the modelling of the advection-dispersion process (Boxall and Guymer, 2007; Julínek and Říha, 2017; Martin et al., 1999; Van Mazijk, 1996; Van Mazijk and Veling, 2005). Comprehensive summaries of experimentally measured data for the longitudinal dispersion coefficient in natural streams are provided by (Zeng and Huai, 2014) and (Wang and Huai, 2016). Attempts to use inverse modelling and genetic algorithms, and to apply neural networks, were made by (Sahay, 2013) and (Ani et al., 2009). Han et al. (2019) carried out an experimental study regarding the relationship between transverse dispersion and diffusion based on the results of dye tests performed in a meandering channel located in the Andong River Experiment Centre (South Korea). The coefficient of transverse dispersion was derived for the meandering channel. The influence of the transverse mixing processes consists of mainly turbulent diffusion and dispersion due to transverse circulations. The transverse dispersion coefficient was studied both under laboratory conditions (Gond et al., 2021; Han et al., 2019; Okoye, 1971; Sayre and Chamberlain, 1964; Seo et al., 2006) and in natural streams (Jeon et al., 2007; Sayre and Yeh, 1973). The tests were performed in order to evaluate the transverse mixing coefficient under different flow conditions. The substance injected into a stream was monitored by measuring tracer concentration in cross-profiles further downstream and calibrating a transverse diffusion model based on the measurements. The tests allowed the inclusion of the influence of hydraulic parameters and channel geometry mainly in the form of empirical formulae for both longitudinal and transverse dispersion coefficients (Aghababaei et al., 2017; Deng et al., 2001; Gond et al., 2021; Huai et al., 2018). Jung et al. (2019) studied the feasibility of using the velocity-based method for calculating the transverse mixing coefficient of the two-dimensional contaminant transport model to substitute the concentration-based method in which the mixing coefficient is calculated from the concentration curves obtained via the tracer experiment. It was found that the meandering of the channel and its tributary controlled the transverse mixing.

A variety of tracers may be applied during experiments (Field, 2003; Pujol and Sanchez-Cabeza, 1999). Tracers should fulfil certain requirements, such as high solubility in water, easy detection, low background concentration in natural streams, conservative behaviour, negligible impact on the aquatic environment, and low cost. From this perspective, fluorescent dyes are the most suitable and most frequently used tracers (Feuerstein and Selleck, 1963). They include Fluorescein, Lissamine FF, Rhodamine B and Rhodamine water tracer (WT) (Mc Cutcheon, 1989). Rhodamine WT is commonly used as a dye tracer, as its properties meet the aforementioned requirements well (Martin et al., 1999; USEPA, 1989). To determine the viability of mapping spatial patterns of dispersion in streams with increased turbidity, Legleiter et al. (2021) performed an experiment with varying dye concentration and turbidity within two tanks while obtaining field spectra, hyperspectral, and RGB (red, green, blue) photos from a small Unoccupied Aircraft System. These data sets were subjected to an optimal band ratio analysis, which revealed significant connections between regionally averaged reflectance and Rhodamine WT dye concentration over four different turbidity levels. Therefore, Rhodamine WT fluorescein tracer was selected for this study, the term "dye test" is used throughout the following text. Burdziakowski et al. (2021) studied dispersion of pollutants within an environment using, tracers Rhodamine WT and uranine. Detection and calculations of tracer concentration was done using unmanned aerial vehicle and a simple digital camera. The method enables obtaining information on the time of arrival, peak concentration, and the dimensions of the dye cloud and its movement in the water environment. Legleiter et al. (2019) based detection of dye cloud progression on a hyperspectral imaging system mounted on UAV. Various detection and graphical techniques are used for the representation of pollution transport both in surface waters (Burdziakowski et al., 2021) and groundwater (Sadeghfam et al., 2022).

Less reliable results may be obtained from empirical formulae applied to the determination of dispersion parameters (namely the coefficient of dispersion) taking into account channel and flow characteristics (Toprak et al., 2004). Empirical formulae are derived based on the physics of dispersion and are validated using data obtained from dye tests. Therefore, the results obtained using such empirical equations should be used for streams with similar conditions to those for which they were derived. Most of the empirical formulae for the determination of dispersion coefficients are based on an equation introduced by Fisher (1967). With respect to the particular hydrodynamic and geometrical characteristics of a stream, modified equations were proposed by Liu (1977) and Seo and Cheong (1998). Deng et al. (2001) implemented the local mixing coefficient into Fischer´s original formula. Longitudinal dispersion identified for natural streams by dye tests was recently published by (Uyigue and Abah, 2020), while empirical formulae for the determination of the coefficient of longitudinal dispersion in small streams were proposed by Oliveira et al. (2017), Murphy et al. (2007), Perrucca et al. (2009), Tealdi et al. (2010) and Sokáč et al. (2020) show the importance of vegetation, having found that mean velocity in channels with vegetation can differ significantly from that found in non-vegetated channels. Analytical solutions for the advection-dispersion equation were provided by, e.g. (Daněček et al., 2002; Van Genuchten and Alves, 1982) under simplified hydrodynamic conditions (e.g. during uniform channel flow). The application of asymmetrical statistical distributions for the simulation of solute transport in streams was studied by (Sokáč et al., 2019).

Advantages and shortcomings of methods for the dispersion coefficient determination is in Table 1.

This study deals with longitudinal and transversal dispersion coefficients obtained via the simplified evaluation of dye tests. The novelty of the evaluation lies in its simplicity, as there is no need for special equipment and advanced analysis procedures during the determination of dye concentration. The results were obtained via the analytical solution of the advection-dispersion equation and compared with both empirical equations and dye tests carried out by various authors.

2. METHODS

The methodology consists of acquiring channel geometry and characteristics (including grain size of the bed) and flow conditions using hydrometric measurement. Further on preparatory works follow including selection of the dye, organisational issues and preliminary analysis of the problem. The dye tests and parallel measurements provide geometric characteristics of the dye cloud at individual time instants. At final evaluation the dispersion coefficients are derived and the method proposed is justified via efficiency coefficients. The methodology is described graphically using the flowchart in Fig. 1.

2.1. Theoretical considerations

The solution to the transport of dissolved matter concerns two separate problems, namely open channel flow and solute transport.

In our case the flow was considered to be steady and uniform during all dye tests. The water depth and average velocity in the considered stream reach were assumed to be approximately constant. This assumption was justified by the measurement of the water level in profiles 0 and D during the tests.

Table 1. Comparison of methods for the dispersion coefficient determination.

Method	Advantages	Disadvantages
	Concentration based methods	
Taking single samples	no special sampling equipment	manual sampling maintenance and transport of single samples laboratory analyses
Continuous measurement (one or more points in a profile)	continuous temporal data, no manipulation with samples	sampling apparatus necessary, need for skilled staff for data transformation and analysis
Arial photo analyses	identification of cloud movement in space and time, analysis of images using GIS techniques	indirect concentration determination with less accuracy, special equipment and licenced pilot necessary
	Dye cloud geometry observation methods	
Manual measurement	identification of cloud movement in space and time, simple technique, no laboratory analyses,	numerous staff for cloud identification, applicable for smaller streams, limited accuracy
Arial photo analyses	description of cloud movement in space and time image analysis with GIS techniques	aerial photography equipment, licenced pilot,
	Stream/river geometry and flow conditions	
Empirical equations	easy to use, no field and laboratory works	appropriate formula for given stream, very limited accuracy

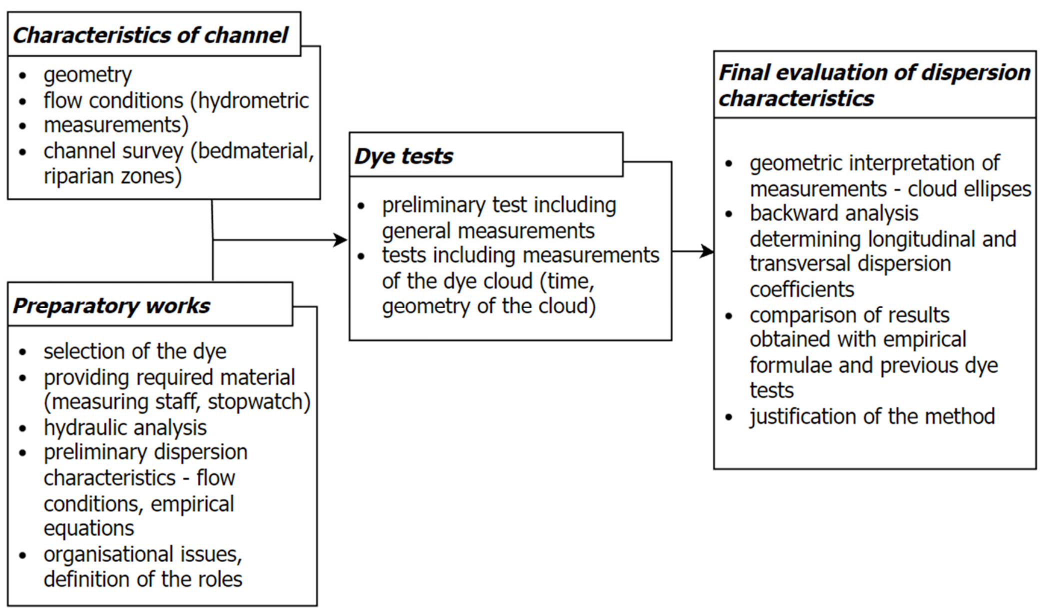

Fig. 1. The flowchart representing proposed methodology.

Traditionally, the problem is treated as one-dimensional (1D) (Ambrose et al., 1996; Van Mazijk, 1996). For a 1D steady uniform open channel flow in the x direction the following holds:

$$Q = Au = AC\sqrt{RJ} \qquad (1)$$

where Q is discharge, A is the flow area, u is the mean velocity in the channel, R is the hydraulic radius, J is the energy slope and C is Chézy velocity coefficient.

The calibration of the simple hydraulic model based on field measurements indicated that flow characteristics during all tests provided only minor changes (see chapters 2.3 and 3.2).

Transport processes in open channels are described by the advection-diffusion equation (Fisher, 1967; Fisher et. al., 1979; Fourier, 1822; Knopman and Voss, 1987), which for the 1D flow (along x direction) and 2D horizontal dispersion (in x, y directions) of conservative matter reads (Singh et al., 2010; Van Mazijk, 1996):

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} - D_x\frac{\partial^2 c}{\partial x^2} - D_y\frac{\partial^2 c}{\partial y^2} = 0 \qquad (2)$$

where c is solute concentration, u is cross-sectional mean velocity, D_x is longitudinal hydrodynamic dispersion (dispersion coefficient) including molecular diffusion and involving the variation of local velocity across the flow profile, D_y is the transversal dispersion coefficient, t is time and x, y are spatial coordinates.

The initial condition expresses the concentration at time $t_0 = 0$,

$$c(x, y, t_0) = c_0(x, y). \tag{3}$$

The Dirichlet boundary condition prescribes concentration in the upstream profile of studied domain $x = 0$:

$$c(0, t) = \bar{c}(t), \tag{4}$$

where $\bar{c}(t)$ is the known concentration. Alternatively, a Neumann boundary condition may be applied in the downstream boundary profile $x = L$:

$$\frac{\partial c(L, t)}{\partial x} = 0. \tag{5}$$

In Eq. (2), the key parameters are flow velocity u and coefficients D_x and D_y, which generally change over time and along the stream. In our case, in order to apply an analytical solution, it is assumed that these parameters are constant in time and along the stream axis.

In this study the longitudinal dispersion coefficient D_x and horizontal transversal dispersion coefficient D_y in the stream were quantified by the analysis of the dye extent identified during the tests and that obtained from the calculation. Due to the small depth of the stream, ideal mixing along the vertical was taken into account. For the case mentioned above, the analytical solution of Eq. (2) holds (Holly and Usseglio-Polatera, 1984; Van Mazijk, 1996):

$$c(x, y, t) = \frac{M_V}{4.\pi.t.h\sqrt{D_x.D_y}} \cdot \exp - \left(\frac{[x - u.t]^2}{4.D_x.t} + \frac{y^2}{4.D_y.t} \right) \tag{6}$$

where M_V is the mass of injected dye and h is the constant water depth, x, y are coordinates (0,0 corresponds to the injection point) and t is time. From Eq. (6), after some manipulation, it results that the constant concentration (e.g. $c = 0$) at a given time is represented by an ellipse.

2.2. Empirical equations

Numerous studies aimed at expressing coefficients D_x and D_y using geometric and hydraulic characteristics of the stream and fluid properties (Seo and Cheong, 1998):

$$D_x = f(\rho, \nu, u, u^*, w, h), \tag{7}$$

where ρ and ν are fluid density and viscosity. The hydraulic characteristics are mean velocity u and shear velocity u^*, and the geometric characteristics are the width of the channel w and water depth h. Dimensional analysis defines the relationship between the longitudinal dispersion coefficient and the above-mentioned characteristics. The influence of the fluid properties in a natural stream is practically negligible (Seo and Cheong, 1998). In such streams the friction losses and all channel irregularities like contractions, expansions, etc. may be included in the shear velocity term:

$$u^* = (g.h.J)^{0.5} \tag{8}$$

where J is the energy line slope.

When approximating $R \approx h$ for shallow flow in a channel the Chézy coefficient C in Eq. (1) may be expressed using the Manning formula

$$C = \frac{1}{n} h^{\frac{1}{6}} \tag{9}$$

and the Manning roughness coefficient n may be expressed from Eqs. (1) and (9) as follows:

$$n = \frac{1}{u} h^{\frac{2}{3}} J^{0.5} \tag{10}$$

Based on these assumptions, Eq. (7) may be rewritten in dimensionless form:

$$D_x = f\left(\frac{u}{u^*}, \frac{w}{h}\right) \tag{11}$$

Relation (11) has been taken into account by various authors, who have expressed the longitudinal dispersion coefficient via empirical equations based on the results of laboratory and field measurements. For the comparison of our results, three empirical formulae have been selected which were derived for conditions similar to those in Lipkovsky Stream (Table 2).

Table 2. Empirical equations for D_x determination.

Author	Empirical equation	
Fisher (1967)	$D_x = 0.011 \frac{u^2 w^2}{h u^*}$	(12)
Wang and Huai (2016)	$D_x = 0.0798\, h u^* \left(\frac{w}{h}\right)^{0.6239} \left(\frac{u}{u^*}\right)^2$	(13)
Oliveira et al. (2017)	$D_x = 0.744 \frac{h^{0.036} u^{1.59}}{u^{*2.22} w^{0.66}}$	(14)

The parameters in the equations in Table 2 are specified in the previous text.

The transversal dispersion coefficient D_y is also related to the hydraulic and geometric characteristics of an open channel (Gond et. al., 2021):

$$D_y = f(u, u^*, w, h, \kappa, S_n), \tag{15}$$

where S_n is stream sinuosity and κ is the coefficient of the flow nonuniformity. In our case, the flow is considered uniform, thus κ is equal to 0. The sinuosity of Lipkovsky Stream is negligible, thus S_n is considered to be 1 for the selected reach. Based on these assumptions, Eq. (15) may be rewritten in dimensionless form:

$$D_y = f\left(\frac{u}{u^*}, \frac{w}{h}\right) \tag{16}$$

Relation (16) has been taken into account by the authors, who expressed the transversal dispersion coefficient via empirical equations based on the results of laboratory and field measurements (Table 3).

Table 3. Empirical equations for D_y determination.

Author	Empirical equation	
Fisher et al. (1979)	$D_y = \alpha_T \, h \, u^*$	(17)
Rutherford (1994)	$D_y = (0.15 \sim 0.30) h \, u^*$	(18)
Chau (2000)	$D_y = 0.18 h \, u^*$	(19)
Huai et al. (2018)	$D_y = h u_* \left[\frac{\left(0.69 \left(\frac{u}{u^*}\right)^{0.47}\right)}{\left(262 + \left(\frac{u}{u^*}\right)^2 - 31.8 \left(\frac{u}{u^*}\right)\right)} + \frac{\left(0.12 \left(\frac{w}{h}\right)^{1.7} \left(\frac{u}{u^*}\right)^{0.35} S_n^{0.395}\right)}{\left(\left(\frac{w}{h}\right) + 0.222 \left(\frac{u}{u^*}\right) - 1.99\right)} \right]$	(20)

The parameters in the equations in Table 3 are specified in the previous text.

In computer codes, the value of the longitudinal dispersion coefficient is usually determined using the following empirical equation (DHI, 2010):

$$D = \alpha h u^* \tag{21}$$

where α is a dispersion factor, u^* is shear velocity and h is water depth.

2.3. The dye test

To determine the transport parameters, four dye tests were carried out at Lipkovsky Stream located in the north of the Czech Republic (Fig. 2). Lipkovsky stream is the right bank tributary of the Ticha Orlice river, the selected reach for the tests is located just upstream of the confluence of both water bodies. The upper part of the catchment is located at the eastern part of the Jeseniky mountains at the Kralicky Sneznik region with naturally preserved area. Close to the Orlice river the land is used for agricultural purposes, Lipkovsky stream was regulated at its lower part. The stream is relatively small and straight (Fig. 3) with steady uniform flow during the tests fitting the above adopted assumptions. Relatively uniform distribution of the velocity along the cross section providing smaller longitudinal dispersion is appropriate for observations and "manual" monitoring of the dye cloud geometry. This also enables to complete generally missing information about dispersion characteristics in small natural streams.

The first test (No. 1) was a trial one to provide preliminary information about the shape and velocity of the dye cloud in the stream. During the three following tests (No. 2 to No. 4), the measurements of the dye cloud dimensions were carried out. The selected reach of Lipkovsky Stream was about 60 m long with constant discharge. The velocity distribution and the discharge in the stream were determined by hydrometric measurements in two profiles, 0 and D (Fig. 4). Obtained value of the discharge was $Q = 0.45$ m^3/s with the mean velocity being approximately $u = 0.63$ m/s. The depth average velocity varied only slightly across the channel width and length (± 0.05 m/s).

The average water depth was $h = 0.25 \pm 0.05$ m, while the width of the stream was about 3.0 to 3.2 m, the mean grain size of the stream bed $D_{50} = 37$ mm. The mean Froude number $Fr = 0.32$ indicates subcritical flow in the stream.

The Rhodamine WT dye was applied as an instantaneous injection in the centreline of the stream in profile 0 (Figs. 4, 5). The mass of injected dye was $M_V = 2$ g in all tests. The centre of the dye cloud was marked by a float placed into the stream together with the dye (Fig. 6).

As the time-dependent spatial sampling of the dye concentration is technically rather difficult, an attempt was made to assess dispersion parameters via the simplified approach of visually determining the extent of the dye cloud progressing along the stream. The arrival time of the cloud front, the position of the cloud centre and the end of the cloud, as well as the width of the central part of the cloud, were simultaneously recorded using measuring staffs during the experiment at selected time intervals. In such a way, the principal axes of ellipses were obtained from which the ellipses were interpreted at individual instants (Fig. 7) during the three tests (No. 2 to No. 4).

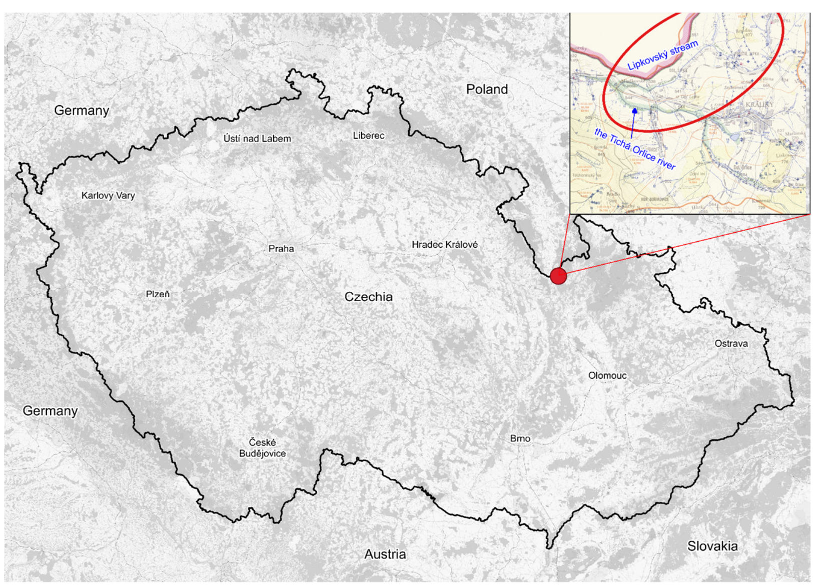

Fig. 2. The map of studied area with the detail of Lipkovsky stream.

Fig. 3. Lipkovsky Stream with the dye (test No. 1).

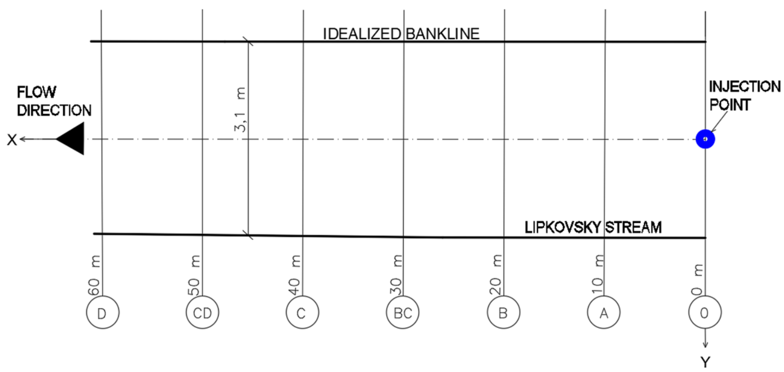

Fig. 4. Schematized stream with injection point and sampling profiles.

Fig. 5. Dye injection.

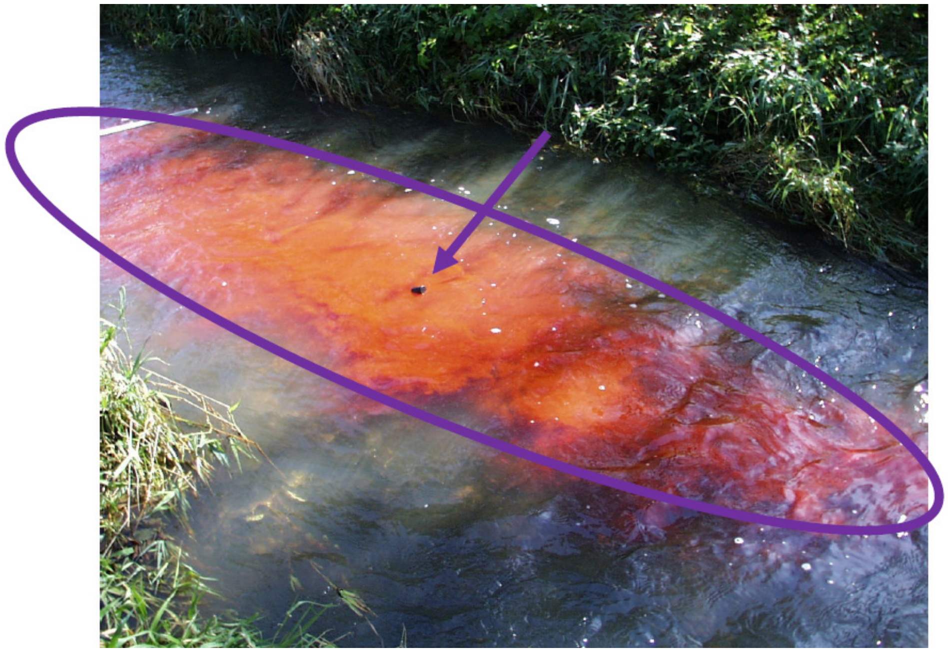

Fig. 6. Approximately elliptical shape of the dye cloud with the float (arrow) at the centre of the cloud.

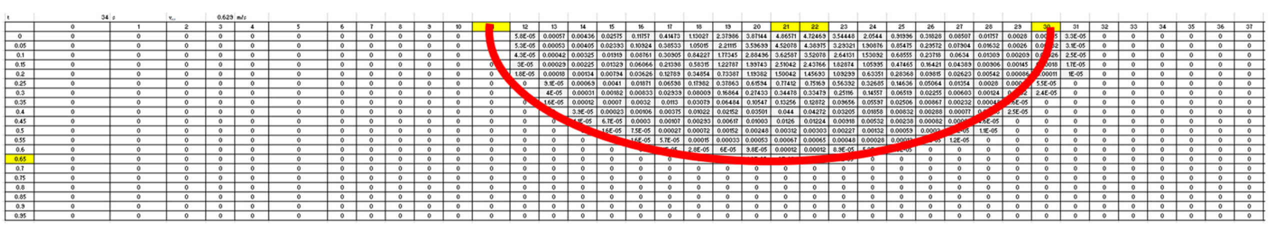

Fig. 7. Example of the resulting calculation obtained from the analytical solution – test No. 3, time 34 s. The red ellipse schematises the observed dye extent.

3. RESULTS

3.1. Measured geometric characteristics of dye clouds

The measurements were related to the profiles according to Fig. 4. The readings of all geometric characteristics were carried out at the instants when the head of the dye cloud reached single profiles. The geometric characteristics were cloud head, centre and tail, and its central width (Table 4). This enabled immediate comparison of resulting distances between individual tests and estimating measurement error counting single decimetres.

From the geometric data in Table 4 the schematic extent of the dye at individual time instants was plotted for each of the three tests (Figs. 8 to 10); the figures also contain calculated results.

3.2. Backward analysis

From Eq. (9) the Manning roughness coefficient n = 0.045 has been determined which fits well the values published e.g. by (Noss and Lorke, 2016) for similar relative roughness.

The dispersion coefficients D_x and D_y were determined by backward analysis carried out by fitting the observed ellipses with the analytical solution obtained from Eq. (6) (Fig. 7). The mass of injected dye $M_V = 2$ g. The trial-and-error method was used. For the backward analysis, the "no dye" zone was represented by a concentration smaller than 10–5 mg/L, as this is the minimum detectable Rhodamine concentration according to Smart and Laidlaw (1977). This value was also considered to be the visibility limit of a similar dye during a test carried out within the study (Julínek and Říha, 2017). The calculation was carried out using Excel software, where dye "ellipses" for individual time instants were clearly visible. They were fitted with ellipses taken from the field observations (Figs. 6, 7). In all tests, $i = 7$ instant measurements of geometric characteristics were carried out.

Using the trial-and-error method, the measured flow velocity u [m/s] was verified based on the location of the centre of the dye clouds where the minimum of the root mean square error (RMSE) was used as the criterion:

$$RMSE_u = \sqrt{\sum_{i=1}^{n=7} \frac{(x_{ci}-\overline{x_{ci}})^2}{n}} = \text{min.} \tag{22}$$

where x_{ci} and $\overline{x_{ci}}$ are the predicted and observed x coordinates of the dye cloud centre (Table 4).

The calibration of the average longitudinal dispersion coefficient D_x [m^2/s] was performed based on the measured locations of the heads and tails of the dye clouds:

$$RMSE_{D_x} = \sqrt{\sum_{i=1}^{n=7} \frac{[(x_{hi}-\overline{x_{hi}})^2+(x_{ti}-\overline{x_{ti}})^2]}{2n}} = \text{min.} \tag{23}$$

where x_{hi} and $\overline{x_{hi}}$ are the predicted and observed x coordinates of the dye cloud heads, and x_{ti} and $\overline{x_{ti}}$ are the predicted and observed coordinates of the tails (Table 4).

Table 4. Overview of dye cloud measurement – observed characteristics of the clouds from the dye tests.

No. of the test	Profile (Fig. 4)	Cloud head		Cloud centre		Cloud tail
		Time	Distance	Distance	Width	Distance
		[s]	[m]	[m]	[m]	[m]
2	0	0	0	0	0	0
		7	5	3.5	0.7	1.6
	A	12.5	10	7.1	0.8	4
	B	24	20	13.5	1.05	7
	BC	36	30	21.5	1.4	12
	C	47	40	30	1.55	17
	CD	58	50	37.5	1.65	26
	D	68	60	44	2	28
3	0	0	0	0	0	0
		6.5	5	2.8	0.5	1.5
	A	12	10	7	0.7	4
	B	22	20	14	0.9	6
	BC	34	30	21.5	1.1	11
	C	45	40	29	1.4	16
	CD	56	50	34.5	1.6	24
	D	67	60	42	1.9	29
4	0	0	0	0	0	0
		5.5	5	3	0.7	2.5
	A	10	10	7.5	0.9	4
	B	21	20	14.5	1.1	7
	BC	33.5	30	21	1.6	12
	C	45.5	40	29.5	1.7	16
	CD	57.5	50	36	1.85	25
	D	68.5	60	43	2.1	31

The determination of the average transversal dispersion coefficient D_y [m^2/s] was based on the central widths of the dye clouds:

$$RMSE_{D_y} = \sqrt{\sum_{i=1}^{n=7} \frac{(y_{ci} - \overline{y_{ci}})^2}{n}} = \text{min}. \tag{24}$$

where y_{ci} and $\overline{y_{ci}}$ are the predicted and observed centre widths of a dye cloud (Table 4).

The resulting mean velocity and dispersion coefficients D_x and D_y are listed in Table 5 together with the corresponding *RMSE*.

Graphical comparisons of the measured and calculated results for the three tests are in Figs. 8 to 10, and an example of the resulting concentration distribution for the times 34 s and 67 s is shown in the 3D diagram in Fig. 11.

Table 5. The resulting mean velocity and dispersion coefficients D_x and D_y and corresponding *RMSE*.

Test No.	u [m/s]	$RMSE_u$ [m]	D_x [m^2/s]	α_x [–]	$RMSE_{D_x}$ [m]	D_y [m^2/s]	α_y [–]	$RMSE_{D_y}$ [m]
2	0.634	1.02	0.051	1.75	2.43	0.00027	0.0093	0.025
3	0.629	0.65	0.057	1.96	2.19	0.00025	0.0086	0.065
4	0.630	0.76	0.052	1.79	1.87	0.00024	0.0083	0.11

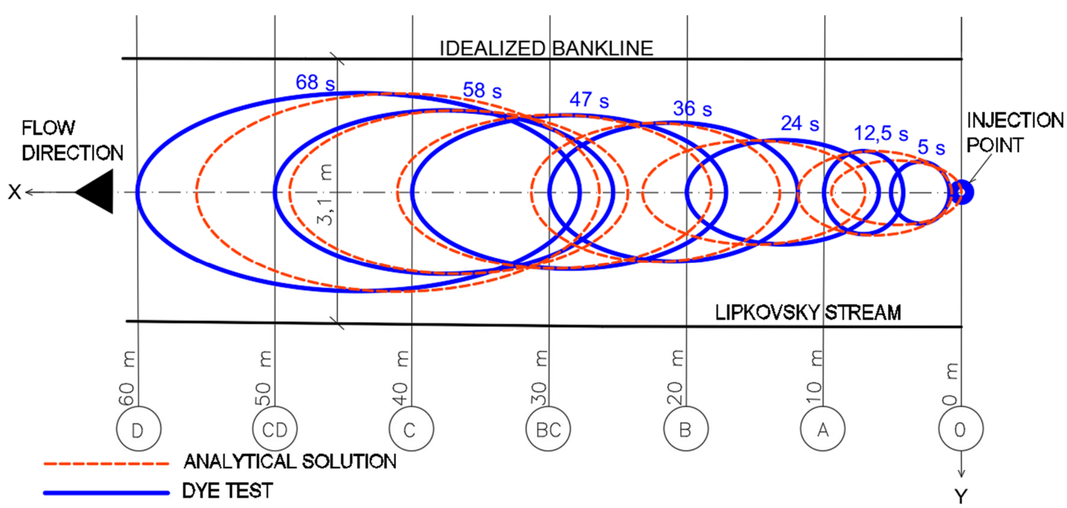

Fig. 8. The results of backward analysis test No. 2 (blue ellipses – dye test, red ellipses – calculation).

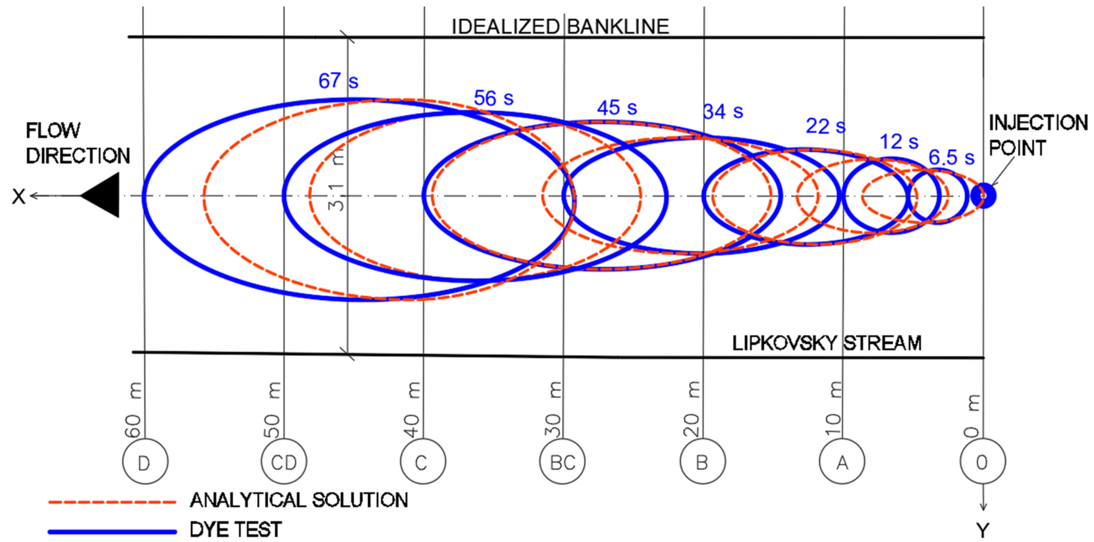

Fig. 9. The results of backward analysis test No. 3 (blue ellipses – dye test, red ellipses – calculation).

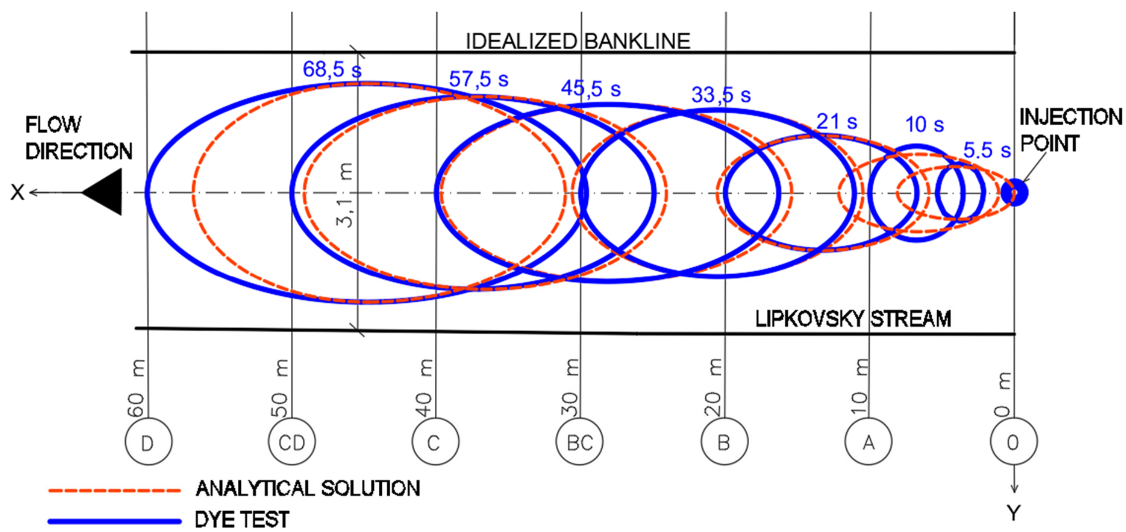

Fig. 10. The results of backward analysis test No. 4 (blue ellipses – dye test, red ellipses – calculation).

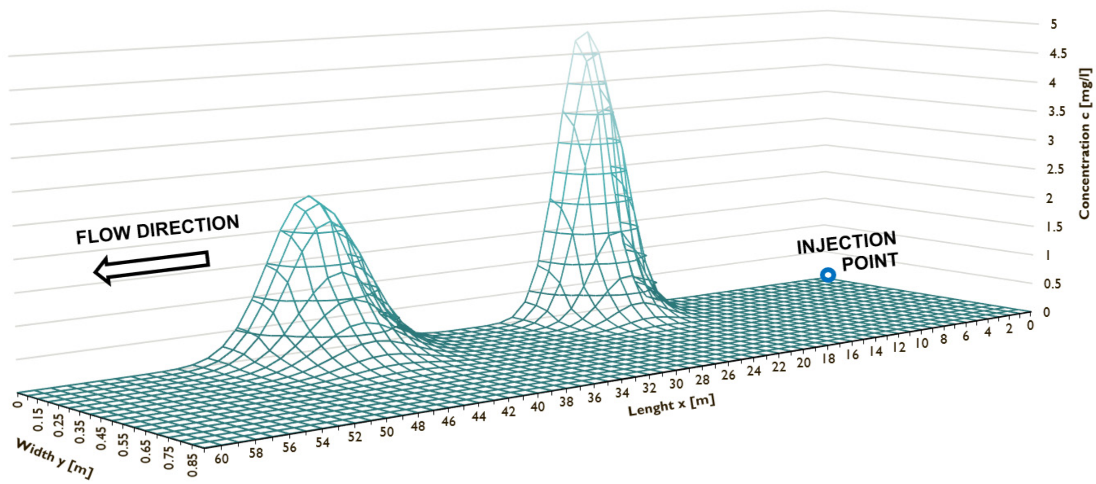

Fig. 11. Spatial interpretation of the concentration distribution obtained from the analytical solution at times 34 s and 67 s for test No. 3 (half of the clouds).

Model efficiency was evaluated using the Nash-Sutcliffe efficiency coefficient (NSE) that determines the relative magnitude of the residual variance compared to the measured data variance (Nash and Surcliffe, 1970). Three coefficients related to individual geometric characteristics were analysed:

$$NSE_u = 1 - \left[\frac{\sum_{i=1}^{7}(x_{ci} - \overline{x_{ci}})^2}{\sum_{i=1}^{7}(x_{ci} - \widehat{x_{ci}})^2}\right], \quad (25)$$

$$NSE_{D_x} = \frac{\left\{1 - \left[\frac{\sum_{i=1}^{7}(x_{hi} - \overline{x_{hi}})^2}{\sum_{i=1}^{7}(x_{hi} - \widehat{x_{hi}})^2}\right]\right\} + \left\{1 - \left[\frac{\sum_{i=1}^{7}(x_{ti} - \overline{x_{ti}})^2}{\sum_{i=1}^{7}(x_{ti} - \widehat{x_{ti}})^2}\right]\right\}}{2}, \quad (26)$$

$$NSE_{D_y} = 1 - \left[\frac{\sum_{i=1}^{7}(y_{ci} - \overline{y_{ci}})^2}{\sum_{i=1}^{7}(y_{ci} - \widehat{y_{ci}})^2}\right]. \quad (27)$$

In Eqs. (25) to (26) where $\widehat{x_{ci}}$ expresses an average of observed x coordinates of the dye cloud centre, $\widehat{x_{hi}}$ and $\widehat{x_{ti}}$ are averages of observed x coordinates of the dye cloud heads and tails and $\widehat{y_{ci}}$ is an average of the centre widths of a dye cloud. The notation of other variables is mentioned above. Results of the Nash-Sutcliffe model efficiency are listed in Table 6.

3.3. Discussion of results

The calibration of dispersion coefficients was performed by minimizing the root mean square error of calculated and measured geometric characteristics of the dye cloud. RMSE for longitudinal characteristics counts single meters, for transversal ones several centimetres (Table 5) which indicates relative error 2–3%. The analysis of predictive skills of the method proposed was carried out using Nash-Sutcliffe efficiency coefficient ranging from 0.9 to 0.998 (Table 6) which for derived dispersion coefficients indicates very good agreement between measured and calculated geometric characteristics.

The obtained results were compared with previously published empirical formulae and dye tests. From the extensive set of formulae and dye experiments published by, e.g. (Wang and Huai, 2016; Zeng and Huai, 2014) and other authors, only results related to approximately similar conditions (geometry and hydraulics of the stream) were chosen.

A comparison of longitudinal dispersion coefficients D_x obtained from our study with results calculated from Eqs. (12), (13), (14) is shown in Table 7. The best fit of D_x in the same order of magnitude provides the formula developed by (Boxall and Guymer, 2007) with an overestimate of about 7 times. Rather worse results (overestimated by about 30 times) are provided by Eq. (12) from (Fisher, 1967). Eq. (14) from (Oliveira et. al., 2017) overestimates our results by about 400 times. The fact that empirical equations may significantly overestimate measured values, sometimes by even more than 200 times, is addressed by, e.g. (Wang and Huai, 2016).

The comparison of obtained longitudinal dispersion coefficients D_x with previous dye tests provides somewhat better results (Table 8, Fig. 12). The best fit is provided by data published by (Glover, 1964; Soares et. al., 2013) with differences starting from 20%.

In case of Lipkovsky Stream, the smaller values of longitudinal dispersion coefficient D_x and factor α_x can be attributed to uniform velocity distribution along the width of the channel. In this study both transversal and longitudinal dispersion were studied, the analysis was carried out in the reach immediately downstream of the injection point where both longitudinal and transversal dispersion take place. In such a case the longitudinal dispersion in streams and flumes may reach very small values. Here longitudinal dispersion is driven mostly due to the differences in point velocities along the vertical. These cause faster dye transport at the water surface with a lag close to the bottom where the velocity drops. This may also be caused by the different hydraulic and geometric characteristics of the streams for which the empirical equations and dye tests were derived.

In the same manner, the transversal dispersion coefficient D_y was compared both with relevant empirical formulae (Table 9) and 28 dye tests taken from the literature (Table 10, Fig. 13). It can be seen that transversal dispersion in Lipkovsky Stream fits both the empirical equations and the results of the dye tests in a much better way than the coefficient of longitudinal dispersion. Empirical equations overestimate our results by about 20 to 40 times. As a rule, lower D_y values correspond to laboratory flume tests and higher values to natural streams. In the case of dye tests, the values of transversal dispersion coefficient D_y are completely comparable.

Table 6. The resulting Nash-Sutcliffe model efficiency coefficient.

Nash-Sutcliffe model test of efficiency *NSE*			
No. of the test	NSE_u	NSE_{D_x}	NSE_{D_y}
2	0.996	0.977	0.994
3	0.998	0.984	0.955
4	0.998	0.983	0.900

Table 7. Comparison with empirical formulae - longitudinal dispersion coefficient D_x and factor α_x.

Method of determination		Longitudinal dispersion coefficient D_x [m²/s]	Longitudinal dispersion factor α_x [–]
Empirical equations	Equation (12)	1.53	55.96
	Equation (13)	0.35	12.70
	Equation (14)	21.82	796.78
This study	Test No. 2	0.051	1.86
	Test No. 3	0.057	2.08
	Test No. 4	0.052	1.90

Table 8. Comparison with dye tests - longitudinal dispersion coefficient D_x and factor α_x.

Author		No.	w [m]	h [m]	u [m/s]	u^* [m/s]	D_x [m²/s]	α_x [–]
Glover (1964)		1	2.40	0.15	0.60	0.059	0.113	12.71
		2	2.40	0.136	0.66	0.026	0.063	17.33
Sayre and Chang (1968)		3	2.35	0.25	0.25	0.075	0.15	7.95
Soares et. al. (2013)		4	3.60	0.39	0.20	0.12	0.36	7.69
		5	1.33	0.32	0.09	0.30	0.10	1.04
		6	1.58	0.32	0.10	0.31	0.13	1.31
		7	1.36	0.19	0.25	0.05	0.51	53.68
		8	2.85	0.27	0.25	0.07	0.23	12.17
Oliveira et. al. (2017)		9	2.4	0.2	0.077	0.11	0.47	21.36
This study	Test No. 2	10	3.1	0.25	0.634	0.110	0.051	1.86
	Test No. 3	11	3.1	0.25	0.629	0.110	0.057	2.08
	Test No. 4	12	3.1	0.25	0.630	0.110	0.052	1.90

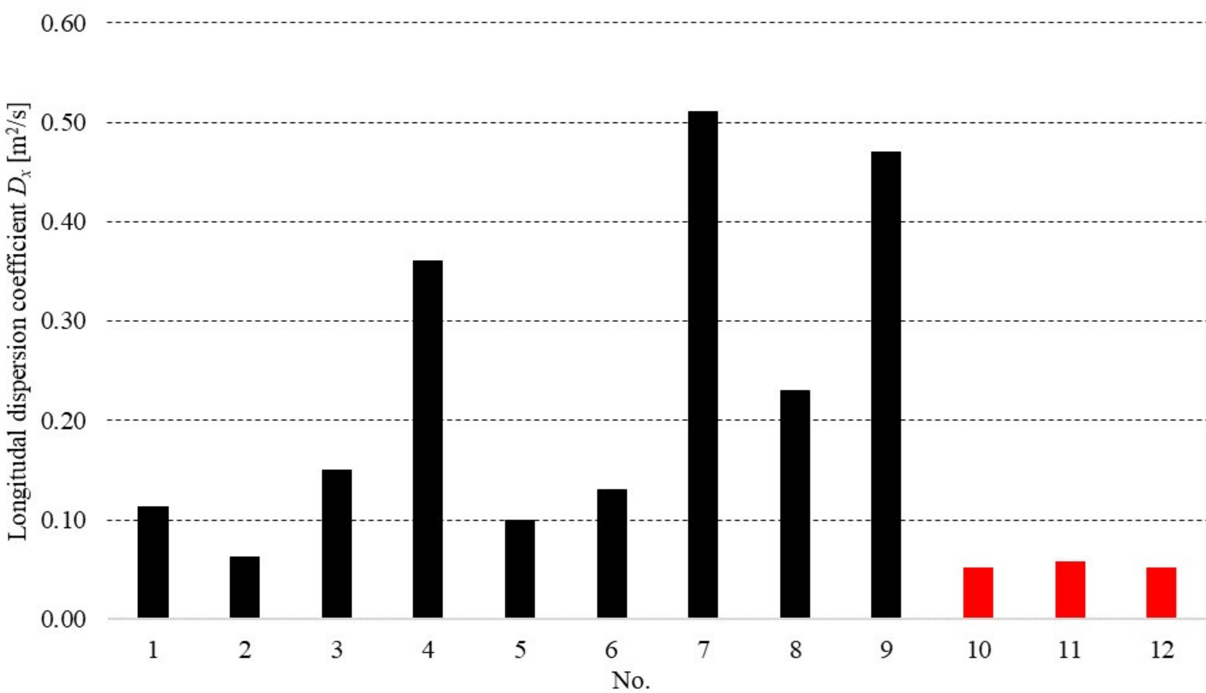

Fig. 12. Comparison of longitudinal dispersion coefficients D_x - this study and dye tests according to Table 8.

Table 9. Comparison with empirical formulae - transversal dispersion coefficient D_y and factor α_y.

Method of determination		Transversal dispersion coefficient D_y [m²/s]	Transversal dispersion factor α_y [–]
Empirical equations	Equation (17)	0.00411	0.142
	Equation (18)	0.00821	0.283
	Equation (19)	0.00493	0.170
	Equation (20)	0.03790	1.305
This study	Test No. 2	0.00027	0.009
	Test No. 3	0.00025	0.009
	Test No. 4	0.00024	0.008

Table 10. Comparison with dye tests - transversal dispersion coefficient D_y and factor α_y.

Author/parameter		No.	w [m]	h [m]	u [m/s]	u^* [m/s]	D_y [m²/s]	α_y [–]
Okoye (1971)		1	0.85	0.02	0.31	0.021	0.00006	0.20
		2	0.85	0.05	0.43	0.023	0.00013	0.11
		3	0.85	0.05	0.43	0.021	0.00013	0.12
		4	0.85	0.05	0.43	0.022	0.00014	0.13
		5	0.85	0.05	0.42	0.021	0.00012	0.11
		6	0.85	0.05	0.42	0.021	0.00015	0.13
		7	0.85	0.11	0.42	0.019	0.00021	0.10
		8	0.85	0.17	0.37	0.018	0.00029	0.09
		9	1.1	0.02	0.33	0.020	0.00008	0.24
		10	1.1	0.03	0.50	0.026	0.00012	0.16
		11	1.1	0.03	0.30	0.017	0.00008	0.17
		12	1.1	0.03	0.32	0.018	0.00009	0.14
		13	1.1	0.05	0.44	0.022	0.00016	0.14
		14	1.1	0.06	0.42	0.022	0.00016	0.14
		15	1.1	0.11	0.39	0.019	0.00028	0.14
		16	1.1	0.17	0.35	0.017	0.00033	0.11
		17	1.1	0.22	0.31	0.014	0.00033	0.11
Prych (1970)		18	1.1	0.04	0.35	0.019	0.00011	0.14
		19	1.1	0.07	0.45	0.021	0.00020	0.15
		20	1.1	0.11	0.46	0.020	0.00036	0.16
		21	1.1	0.04	0.37	0.037	0.00020	0.14
		22	1.1	0.06	0.46	0.040	0.00035	0.14
Rishnappan and Lau (1977)		23	0.6	0.04	0.34	0.020	0.00014	0.17
		24	0.6	0.04	0.31	0.018	0.00011	0.16
		25	0.6	0.05	0.30	0.017	0.00014	0.17
		26	0.6	0.04	0.34	0.020	0.00013	0.16
		27	0.6	0.04	0.31	0.018	0.00011	0.16
		28	0.6	0.05	0.30	0.017	0.00014	0.16
This study	Test No. 2	29	3.1	0.25	0.634	0.110	0.00027	0.009
	Test No. 3	30	3.1	0.25	0.629	0.110	0.00025	0.009
	Test No. 4	31	3.1	0.25	0.630	0.110	0.00024	0.008

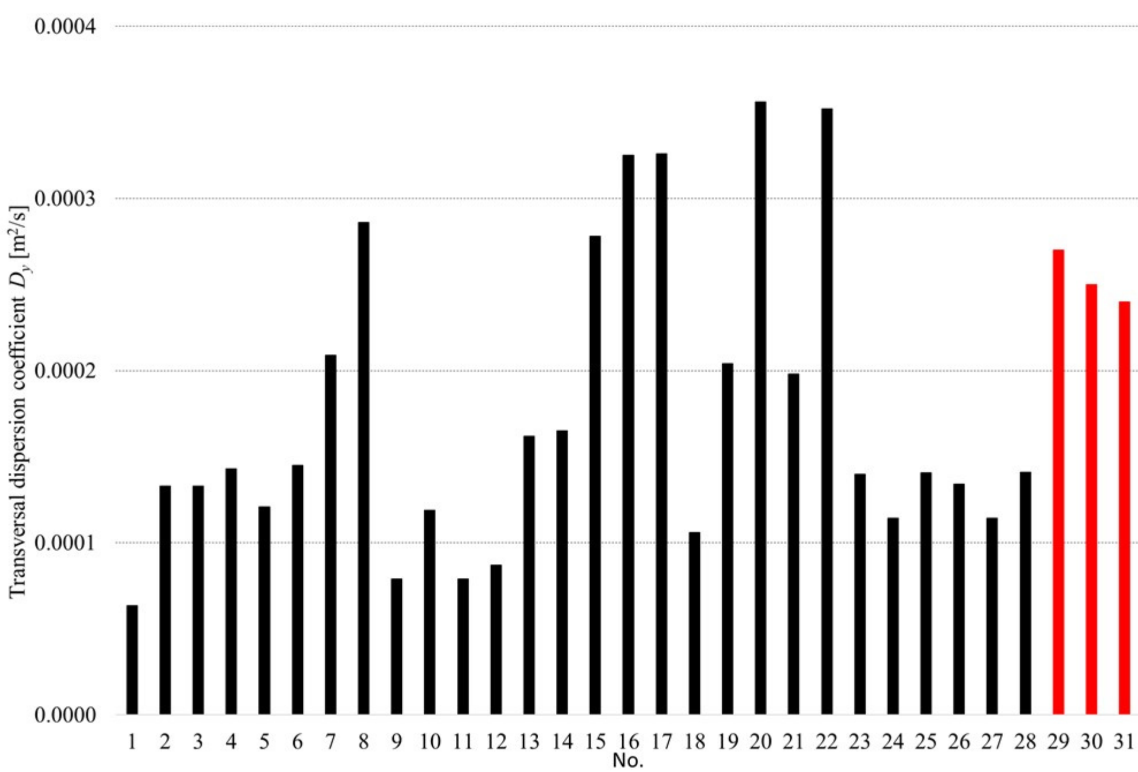

Fig. 13. Comparison of transversal dispersion coefficients D_y - this study and dye tests according to Table 10.

4. CONCLUSIONS

A simple method for the estimation of longitudinal and transversal dispersion based on the visual tracing of dye clouds in streams was proposed and tested. The evaluation of the geometric dimensions of the dye clouds was carried out via the analytical solution (6) of the advection-dispersion Equation (2). The method was verified at the relatively small Lipkovsky Stream along a 60 m long straight reach with approximately uniform velocity distribution also along the constant stream width. The advantage of the method is its simplicity: it can be employed without the need for special equipment and advanced analysis procedures for the determination of dye concentration.

The resulting D_x and D_y (α_x and α_y) values were compared with values determined by empirical formulae and with published values obtained from dye tests. Studies with similar channel and flow parameters were selected. The comparison of the values from dye tests at Lipkovsky Stream (D_x = 0.051–0.057 m^2/s, D_y = 0.00024–0.00027 m^2/s) with empirical formulae and other tracer studies indicated significant overestimating of the mentioned values of D_x, which can be attributed to the uniform velocity distribution along the width of Lipkovsky Stream. Much better agreement was achieved for D_y, namely with values obtained from dye tests carried out in laboratory flumes and small streams. Relative error between calculated and measured geometric characteristics counts 2–3%, Nash - Sutcliffe efficiency coefficient ranges between 0.9 and 0.998 which signifies good predictive skills of the proposed method.

Even though the simplified method was only tested at a small stream with a width of 3.1 m, the authors believe that it can also be applied for the evaluation of dye tests or pollution spills in larger watercourses such as navigation or irrigation canals, regulated rivers, etc., where the extent and dimensions of the dye cloud may be monitored and processed by drones combined with GPS techniques.

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REFERENCES

Aghababaei, M., Etemad-Shahidi, A., Jabbari, E., Taghipour, M., 2017. Estimation of transverse mixing coefficient in straight and meandering streams. Water Resource Management, 31, 3809–3827. https://doi.org/10.1007/s11269-017-1708-4

Ambrose, R.B., Barnwell, T.O., McCutcheon S.C., Williams, J.R., 1996. Computer models for water-quality analysis. Chapter 14. In: Mays, L.W. (Ed.): Water Resources Handbook. McGraw-Hill Companies, New York, NY, pp. 14.1–14.28.

Ani, E.C., Wallis, S., Kraslawski, A., Agachi, P.S., 2009. Development, calibration and evaluation of two mathematical models for pollutant transport in a small river. Environmental Modelling & Software, 24, 10, 1139–1152. https://doi.org/10.1016/j.envsoft.2009.03.008

Azamathulla, H.M., Wu, F.C., 2011. Support vector machine approach for longitudinal dispersion coefficients in natural streams. Appl. Soft Computing. J., 11, 2, 2902–2905. https://doi.org/10.1016/j.asoc.2010.11.026

Boxall, J.B., Guymer, I., 2007. Longitudinal mixing in meandering channels: New experimental data set and verification of a predictive technique. Water Research, 41, 341–354.

Burdziakowski, P., Zima, P., Wielgat, P., Kalinowska, D., 2021. Tracking fluorescent dye dispersion from an unmanned aerial vehicle. Sensors, 21, 11, 3905. https://doi.org/10.3390/s21113905

Brown, L.C., Barnwell, T.O., 1987. The Enhanced Stream Water Quality Models. QUAL2E - UNCAS - Documentation and User Manual. Athens, USA, 188 p.

Chapra, S.C., 1997. Surface Water Quality Modeling. McGraw Hill, USA, 1997, 844 p.

Chau, K.W., 2000. Transverse mixing coefficient measurements in an open rectangular channel. Advances in Environmental Research, 4, 287–294.

Crowder, R.A., Pepper A.T., Whitlow, C., Sleigh, A., Wright, N., Tomlin, C., 2004. Benchmarking hydraulic river modelling software packages. R&D Technical Report. Defra/Environment Agency. Great Britain, 40 p.

Daněček, J., Ryl, T., Říha, J., 2002. Determination of longitudinal hydrodynamic dispersion in water courses with solution of Fischer's integral. J. Hydrology Hydromech., 50, 2, 104–113.

Deng, Z.Q., Singh, V.P., Bengtsson, L., 2001. Longitudinal dispersion coefficient in straight rivers. J. Hydraulic Eng., 127, 11, 919–927.

DHI, 2010. MIKE 11 Reference Manual. Danish Hydraulic Institute. Hørsholm.

Feuerstein, D.L., Selleck, R.E., 1963. Fluorescent tracers for dispersion measurement. Journal of the Sanitary Engineering Division, 89, 4, 1–22.

Field, M., 2003. Tracer-Test Planning Using the Efficient Hydrologic Tracer-Test Design (EHTD) Program 2005. U.S. Environmental Protection Agency. Washington DC, EPA/600/R-03/034B, 175 p.

Fischer, H.B., 1967. The mechanics of dispersion in natural streams. Journal of the Hydraulic Division. Proceedings of ASCE, 93, HY6, 187–216.

Fischer, H.B., List, J., Koh, C., Imberger, J., Brooks, N.H., 1979. Mixing in Inland and Coastal Waters. Academic Press. New York, 483 p.

Fourier, J., 1822. Théorie analytique de la chaleur. (The analytical theory of heat). Firmin Didot Père et Fils, Paris, 639 p.

Glover, R.E., 1964. Dispersion of dissolved or suspended materials in flowing streams. US Geological Survey. Professional Paper Vol. 433. https://doi.org/10.3133/pp433b

Gond, L., Mignot, E., Coz, J.L., Kateb, L., 2021. Transverse mixing in rivers with longitudinally varied morphology. Water Resources Research, 57, e2020WR029478. https://doi.org/10.1029/2020WR029478

Halmova, D., Miklanek, P., Pekar, J., Pramuk, B., Pekarova, P. 2014. Longitudinal dispersion coefficient in natural streams in Slovakia. Bodenkultur, 65, 3–4, 23–29.

Han, E.J., Kim, Y.D., Baek, K.O., Seo, I.W., 2019. Relation between transverse dispersion and diffusion at meandering channel in two - dimensional mixing based on tracer tests. Environmental Earth Sciences, 78, 712. https://doi.org/10.1007/s12665-019-8710-5

HEC-RAS, 2022. Hydrologic Engineering Center's River Analysis System. USACE. http://www.hec.usace.army.mil (last access: 9 September 2022).

Holly, F.M., Usseglio-Polatera, J.M., 1984. Dispersion simulation in two-dimensional tidal flow. Journal of Hydraulic Engineering, 110, 7, 905–926.

Huai, W., Shi, H., Yang, Z, Zeng, Y., 2018. Estimating the transverse mixing coefficient in laboratory flumes and natural rivers. Water Air Soil Pollut., 229, 252. https://doi.org/10.1007/s11270-018-3893-z

Jeon, T.M., Baek, K.O., Seo, I.W., 2007. Development of an empirical equation for the transverse dispersion coefficient in natural streams. Environ. Fluid Mech., 7, 317–329, https://doi.org/ 10.1007/s10652-007-9027-6

Julínek, T., Říha, J., 2017. Longitudinal dispersion in an open channel determined from a tracer study. Environ. Earth Sci., 76, 592. https://doi.org/10.1007/s12665-017-6913-1

Jung, S.H., Seo, I.W., Kim, Y.D., Park, I., 2019. Feasibility of velocity-based method for transverse mixing coefficients in river mixing analysis. J. Hydraul. Eng., 145, 11. https://doi.org/10.1061/(ASCE)HY.1943-7900.0001638

Kim, D., 2012. Assessment of longitudinal dispersion coefficients using Acoustic Doppler Current Profilers in large river. Journal of Hydro-environment Research, 6, 1, 29–39. https://doi.org/10.1016/j.jher.2011.06.001

Knopman, D.S., Voss, C.I., 1987. Behaviour of sensitivities in the one-dimensional advection-dispersion equation: implications for parameter estimation and sampling design. Water Resources Research, 23, 253–272.

Legleiter, C.J., McDonald, R.R., Nelson, J.M., Kinzel, P.J., Perroy, R.L., Baek, D., Seo, I.W., 2019. Remote sensing of tracer dye concentrations to support dispersion studies in river channels. J. Ecohydraulics, 4, 131–146.

Legleiter, C.J., Manley, P.V., Erwin, S.O., Bulliner, E.A., 2021. An experimental evaluation of the feasibility of inferring concentrations of a visible tracer dye from remotely sensed data in turbid rivers. Sensors, 12, 1, 57. https://doi.org/10.3390/rs12010057

Leibundgut, C., Speidel, U., Wiesner, H., 1993. Transport processes in rivers investigated by tracer experiments. In: Proceedings of the Yokohama Symposium Tracers in Hydrology. IAHS Publ. no. 215, pp. 211–217.

Liu, H., 1977. Predicting dispersion coefficient of streams. Journal of Environmental Engineering Division, 103, 1, 59–69.

Murphy, E., Ghisalberti, M., Nepf, H., 2007. Model and laboratory study of dispersion in flows with submerged vegetation. Water Resources, 43, 5. https://doi.org/10.1029/2006WR005229

Martin, J.L., McCutcheon, S.C., Schottman, R.W., 1999. Hydrodynamics and Transport for Water Quality Modeling. Boca Raton. Lewis Publishers. N. Y., 795 p.

Mc Cutcheon, S.C., 1989. Water Quality Modelling. Vol.-I. Transport and Surface Exchanges in Rivers. CRC Press Inc., pp. 103–110.

Nadal, A.F., Lozada, J.M.D., Medina, G.P.B., Moro, K.M., Melchiorre, M., Larrosa, N.B., 2021. Longitudinal scattering coefficient obtained by hydroacoustic measurement and by conservative tracer. Revista de Ciencia y Tecnología, 23, 36, p. 53–60.

Nash, J. E., Sutcliffe, J.V., 1970. River flow forecasting through conceptual model. Part 1 - A discussion of principles. Journal of Hydrology, 10, 282–290.

Noss, C., Lorke, A., 2016. Roughness, resistance, and dispersion: Relationships in small streams. Water Resour. Res., 52, 2802–2821. DOI: 10.1002/2015WR017449

Okoye, J.K., 1971. Characteristics of transverse mixing in open channel flows. Ph.D. Thesis. California Institute of Technology, 269 p.

Oliveira, V.V., Mateus, M.V., Gonçalves, J.C.S.I., Utsumi, A.G., Giorgetti, M.F., 2017. Prediction of the longitudinal dispersion coefficient for small watercourses. Acta Scientiarum Technology, 39, 3, 291–299. https://doi.org/10.4025/actascitechnol.v39i3.29397

Park, I., Seo, I.W., Shin, J., Song, C.G., 2020. Experimental and numerical investigations of spatially-varying dispersion tensors based on vertical velocity profile and depth-averaged flow field. Adv. Water Resour., 142, 8. https://doi.org/10.1016/j.advwatres.2020.103606

Perucca, E., Camporeale, C., Ridolfi, L., 2009. Estimation of the dispersion coefficient in rivers with riparian zone. Advances in Water Resources, 32, 78–87.

Pujol, L.I., Sanchez-Cabeza, J.A., 1999. Determination of longitudinal dispersion coefficient and velocity of the Ebro River waters (Northeast Spain) using tritium as a radiotracer. Journal of Environmental Radioactivity, 45, 39–57.

Prych, E.A., 1970. Effects of density differences on lateral mixing in open-channel flows. California Institute of Technology, 243 p. DOI: 10.7907/WEDF-B722

Riahi-Madvar, H., Ayyoubzadeh, S.A., Khadangi, E., Ebadzadeh, M.M., 2009. An expert system for predicting longitudinal dispersion coefficient in natural streams by using ANFIS. Expert Systems with Applications, 36, pp. 8589–8596.

Rishnappan, B.G., Lau Y.L., 1977. Transverse mixing in meandering channels with varying bottom topography. Journal of Hydraulic Research, 15, 4, 351–370.

Rutherford, J.C., 1994. River Mixing. Wiley, 347 p. ISBN-0-471-94282-0.

Sadeghfam, S., Bagheri, A, Razzagh, S., Nadir, A A., Vadiati, M., Senapathi, V., Sekar, S., 2022. Hydrochemical analysis of seawater intrusion by graphical techniques in coastal ‘aquifers to delineate vulnerable areas. In: Groundwater Contamination in Coastal Aquifers, pp. 91–104.

Sahay, R.R., 2013. Predicting longitudinal dispersion coefficients in sinuous rivers by genetic algorithm. J. Hydrol. Hydromech., 61, 3, 2013, 214–221. DOI: https://doi.org/10.2478/johh-2013-0028

Sayre, W.W., Chamberlain, A.R., 1964. Exploratory Laboratory Study of Lateral Turbulent Diffusion at the Surface of an Alluvial Channel. U.S. Geological Survey. Open file report 196, 19 p.

Sayre, W.W., Chang, F.M.A, 1968. laboratory investigation of open-channel dispersion processes for dissolved, suspended, and floating dispersants. Geological Survey Professional Paper, USDI, 433-E, 75 p. https://doi.org/10.3133/pp433E.

Sayre, W.W., Yeh, T., 1973. Transverse mixing characteristics of the Missouri River downstream from the Cooper Nuclear Station. Technical Report. Iowa Institute of Hydraulic Research, IIHR-145, TRN: 77-016002, 46 p.

Seo, I.W., Cheong, T.S., 1998. Predicting longitudinal dispersion coefficient in natural streams. J. Hydraul. Eng., 124, 1, 25–32.

Seo, I.W., Baek, K., Jeon, T.M., 2006. Analysis of transverse mixing in natural streams under slug tests. Journal of Hydraulic Research., 44, 3, 350–362, https://doi.org/10.1080/00221686.2006.9521687

Shin, J., Seo, I.W., Baek, D., 2020. Longitudinal and transverse dispersion coefficients of 2D contaminant transport model for mixing analysis in open channels. Journal of Hydrology, 583, 1200, 124302. https://doi.org/10.1016/j.jhydrol.2019.124302

Singh, M.K., Singh, P., Singh, V.P., 2010. Analytical solution for two-dimensional solute transport in finite aquifer with time dependent source concentration. Journal of Engineering Mechanics, 136, 10, 1309–1315. https://doi.org/10.1061/(ASCE)EM.1943-7889.0000177

Smart, P.L., Laidlaw, I. M.S., 1977. An evaluation of some fluorescent dyes for water tracing. Water Resources Research, 13, 1, 15–33.

Soares, P.A., Pinheiro, A., Zucco, E., 2013. Determinação do coeficiente de dispersão longitudinal em rios. Revista de Gestão de Água da América Latina, 10, 2, 27–36.

Sokáč, M., Velísková, Y., Gualtieri, C., 2019. Application of asymmetrical statistical distributions for 1D simulation of solute transport in streams. Water (Switzerland), 11, 10, 2145.

Sokáč, M., Schügerl, R., Velísková, Y., Dulovičová, R., 2020. Influence of aquatic vegetation on dispersive parameters as a part of hydrodynamic conditions in natural streams. Earth and Environmental Science, 609, 115. https://doi.org/10.1088/1755-1315/609/1/012037

Tealdi, S., Camporeale, C., Perruca R., 2010. Longitudinal dispersion in vegetated rivers with stochastic flows. Advances in Water Resources, 33, 562–571.

Toprak, Z.F., Sen, Z., Savci, M.E., 2004. Comment on "Longitudinal dispersion coefficients in natural channels". Water Research, 38, 13, 3139–3143. DOI: 10.1016/j.watres.2003.08.004

USEPA, 1989. Exposure Factors Handbook. U.S. Environmental Protection Agency. Office of Health and Environmental Assessment. EPA/600/8-89/043. Washington. D.C., 1216 p.

Uyigue, L., Abah, M.A., 2020. Evaluation of the longitudinal dispersion coefficient for Dor-Nwezor-Bodo River using tracer experiment and theoretical measurement. Res. Rev. J. Eng. Technol., 9, 1, 6–13.

Van Genuchten, M.T., Alves, W.J., 1982. Analytical Solutions of the One-dimensional Convective-dispersive Solute Transport Equation. Technical Bulletin No. 1661. United States Department of Agriculture. Washington, 149 p.

Van Mazijk, A, 1996. One-dimensional approach of transport phenomena of dissolved matter in rivers. Communication on hydraulic and geotechnical engineering. FCE TU Delft. Report No. 96-3, 310 p.

Van Mazijk, A., Veling, E.J.M., 2005. Tracer experiments in the Rhine Basin: evaluation of the skewness of observed concentration distributions. Journal of Hydrology, 307, 60–78.

Veliskova, Y., Kohutiar, J., 1992. Two-dimensional model of dispersion in natural channels. J. Hydrol. Hydromech., 40, 5, 409–424.

Wang, Y., Huai, W., 2016. Estimating the longitudinal dispersion coefficient in straight natural rivers. J. Hydraul. Eng., 142, 11. https://doi.org/10.1061/(ASCE)HY.1943-7900.0001196

Zeng, Y.H., Huai, W.X., 2014. Estimation of longitudinal dispersion coefficient in rivers. Journal of Hydro-environment Research, 8, 2–8.

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